Combinatorics. Basics of probability

Probability theory

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Higher School of Economics

DSBA 221
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Seminar Overview

1. Organization
   - Grades
   - Learning
   - Penalties
2. Basics of probability
   - Reminder of definitions
   - Problems

Grades formula

First half-year

$$\begin{aligned} &\text{Home Assignments of Semester 1} \cdot 20\% \\ &\qquad + \\ &\text{Fall Midterm} \cdot 45\% \quad = \quad \text{Semester 1 Score} \\ &\qquad + \\ &\text{Winter Exam} \cdot 35\% \end{aligned}$$

Grades formula

Second half-year

$$
\begin{aligned}
&\text{Fall Midterm} \cdot 20\% \\
&+ \\
&\text{Winter Exam} \cdot 15\% \\
&+ \\
&\text{Home Assignments of Semester 1} \cdot 10\% \\
&+ \qquad = \quad \text{Final Score} \\
&\text{Spring Midterm} \cdot 25\% \\
&+ \\
&\text{Home Assignments of Module 3} \cdot 15\% \\
&+ \\
&\text{Home Assignments of Module 4} \cdot 20\%
\end{aligned}
$$

Quizzes

- Quizzes are being written in 10 minutes at the start of each class.
- Quizzes consist of 2 problems, 2 pts each.
- Their scores are taken into account within Home Assignments:

$$\begin{aligned}\text{HA Module X Score} = {} & \text{HA Module X Score} \cdot 90\% \\ & + \\ & \text{Quizzes Module X Score} \cdot 10\%\end{aligned}$$

Keeping track of grades

Pass your home assignments in a Google classroom:

Google Classroom course code

chhaljz

Follow your grades for home works and quizzes in Google spreadsheet:

Google spreadsheet with HA and quizzes scores

Click Me

Asking questions

Write to me at any time before 10 PM in Telegram. I always answer back =)

My Telegram account

@tonyliveswell

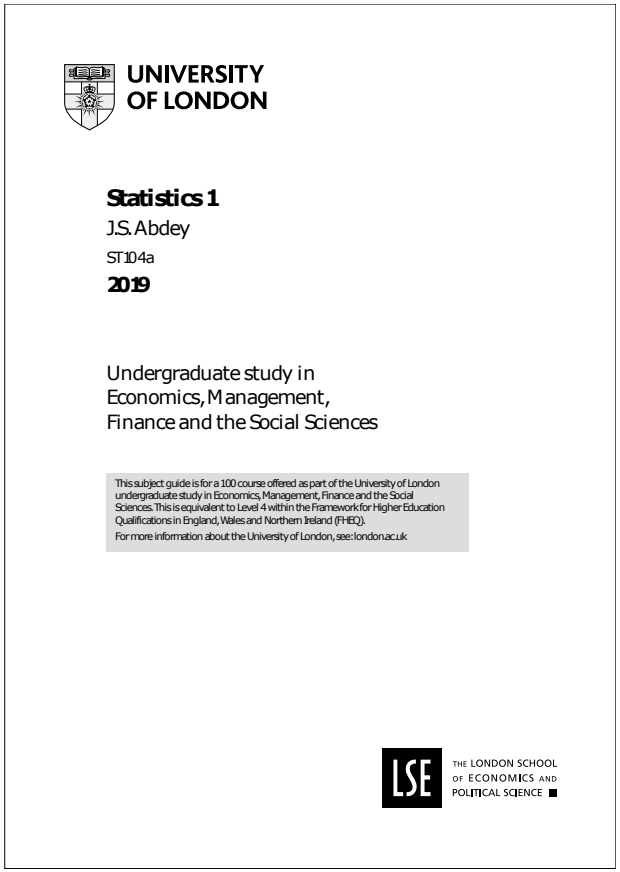
Your Teaching Assistant for this course is Iskander Sergazin. You can also write to him in Telegram:

Iskander's Telegram account

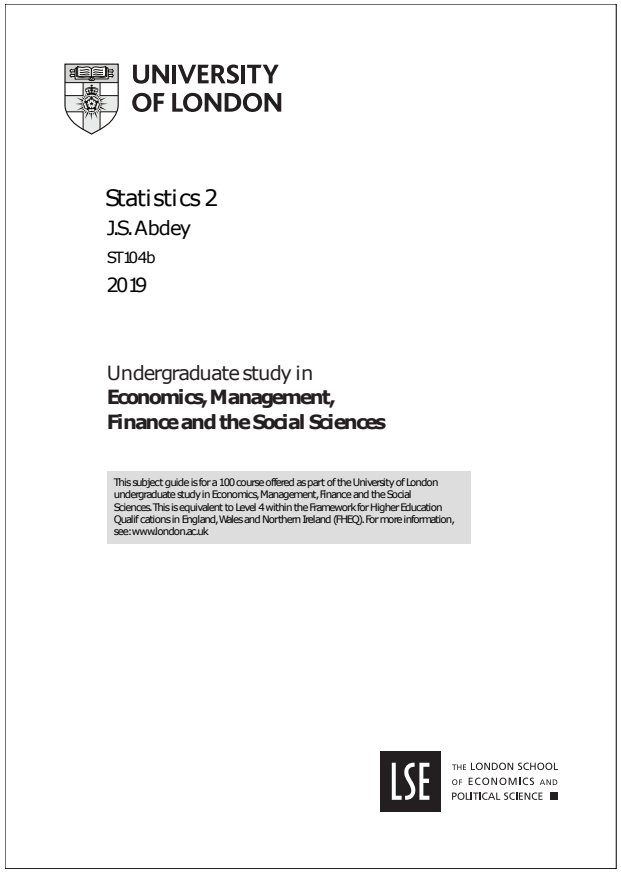
@I_s_k_aR

What should I read?

Must read



(a) Statistics 1



(b) Statistics 2

Figure: University of London study guides by J.S. Abdey.

What should I read?

Handbook after the course

Figure: “Visualizing mathematical statistics” by M.B. Lagutin.

Penalties

- Home assignments will be zerorized if CTRL+C/CTRL+V is noticed.
- As in a good autocratic environment, we have presumption of guilt in this course. So a student has to prove that they are innocent =)
- A student can prove their knowledge after the class, taking extra problems to solve in front of the whiteboard.
- Each day off the deadline in passing the home assignment decreases maximum earnable points by 20%.

Classical definition of probability

- ω_i – outcome of an experiment.
- $\Omega = \{\omega_1, \ldots, \omega_n\}$ – sample space of an experiment if outcomes are:

  1. collectively exhaustive,
  2. mutually exclusive,
  3. equiprobable (only in classical definition).

Example

For rolling a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$.
For tossing a coin twice: $\Omega = \{HH, HT, TH, TT\}$.

- A – arbitrary subset of Ω.
- $\mathsf{P}(A) = \dfrac{|A|}{|\Omega|}$.

Problem 1

Suppose that a class of 100 students consists of four subgroups, in the following proportion:

	Men	Women
Taking economics	17%	38%
Not taking economics	23%	22%

What is the chance that a randomly chosen student is:

1. a woman?
2. taking economics?
3. a man or taking economics?
4. a woman and taking economics?

Problem 2

20 families live in a neighborhood of Wolfenstein Castle: 4 have 1 child, 8 have 2 children, 5 have 3 children, and 3 have 4 children. If we meet a local child near the castle, what are the probabilities $p_1,\ p_2,\ p_3,\ p_4$, that the child comes from a family with 1, 2, 3, 4 children?

Problem 3

Suppose a word is picked at random from this sentence.

1. What is the sample space of this random experiment?
2. Find the probability that:
   1. the word has at least 4 letters;
   2. the word contains at least 2 vowels;
   3. the word contains at least 4 letters and at least 2 vowels.

Problem 4

Find unions and intersections of the following events. In which case one event is a subset of the other?

1. $A = \{1, 2, 5, 6\},\ B = \{1, 5\}$.
2. $A = \{\text{Ann, Mary, Mike}\},\ B = \{\text{Tom, Mike, John}\}$.
3. $A = \{\text{Moscow, London, Paris}\},\ B = \{\text{Paris, Berlin, Tokyo}\},$ $C = \{\text{Tokyo, Rome}\}$.

Problem 5

In a group of students (none of whom are from DSBA) 25% smoke hookah, 60% drink alcohol, and 15% do both. What fraction of students have at least one of these bad habits?

Problem 6

In a group of 320 graduates of the School of Witchcraft and Wizardry (all of whom are either witches or wizards) only 160 went to college, but 100 of 170 wizards did. What is the probability that a witch chosen at random from the group of school graduates did not go to college?

Problem 7

1. If A and B are mutually exclusive events with probabilities of 0.6 and 0.2 respectively, then what is the probability of A or B occurring?
2. If $\mathsf{P}(A) = 0.2,\ \mathsf{P}(B) = 0.3$, and $\mathsf{P}(A \cap B) = 0.1$, then what is $\mathsf{P}(A \cup B)$?
3. If $\mathsf{P}(A) = 0.4,\ \mathsf{P}(B) = 0.5$, and $\mathsf{P}(A \cup B) = 0.7$, then what is $\mathsf{P}\left(\overline{A \cap B}\right)$?

Problem 8

A survey of the houses in an old residential area found 30% with holes in the roof, 40% with broken windows, and 25% with the both problems.

1. What is the proportion of houses with one or the other (or both) problems?
2. What is the proportion of houses with holes in the roof but without broken windows?
3. What is the proportion of houses with exactly one of these problems?
4. What is the proportion of houses with none of these problems?

Problem 9

Suppose that in families with three children births are independent, and the probability of a boy on each birth is 52 %. Use the table

Outcome	BBB	BBG	BGB	BGG	GBB	GBG	GGB	GGG
Probability	0.14	0.13	0.13	0.12	0.13	0.12	0.12	0.11

and find the chance that in a family of three children, there will be:

1. exactly 2 girls;
2. at least two girls;
3. at least one child of each sex;
4. the middle child being opposite in sex to the other two.

Problem 10

You take 3 cards from a deck of 36 cards. After you take each card you return it to the deck and shuffle the deck.

1. What is the set of elementary outcomes and what are their probabilities?
2. What is the probability to get (queen, king, ace) ?
   (The first card is a queen, the second card is a king, the third card is an ace).
3. What is the probability to get (king, king, ace) ?

Problem 11

You take 3 cards from a deck of 36 cards. After you take each card you do not return it to the deck. Answer (1), (2), (3) from the previous problem.

Problem 12

Suppose we roll a red die and a green die. What is the probability that the number on the red die is larger than the number on the green die?

Problem 13

Two dice are rolled. Find the probability that

1. the two numbers will differ by 1 or less;
2. the maximum of the two numbers will be 5 or larger.

Problem 14

In Galileo's time people thought that when three dice were rolled, a sum of 9 points and a sum of 10 points had the same probability, since each could be obtained in 6 ways:

$$9: \quad 1+2+6,\ 1+3+5,\ 1+4+4,\ 2+2+5,\ 2+3+4,\ 3+3+3.$$

$$10: \quad 1+3+6,\ 1+4+5,\ 2+4+4,\ 2+3+5,\ 2+2+6,\ 3+3+4.$$

1. Compute the probability of the event $1+2+6$ (one point on one die, two points on the other, and 6 points on the remaining die).
2. Compute the probability of the event $2+4+4$ (two points on one die, and four points on each of the remaining dice).
3. Find the probabilities of the events $A =$ "A total of 9 points on three dice", and $B =$ "A total of 10 points on three dice".

Problem 15

Let A and B be events with probabilities $\mathsf{P}(A) = \frac{3}{4}$ and $\mathsf{P}(B) = \frac{1}{3}$.
Show that

$$\frac{1}{12} \leq \mathsf{P}(A \cap B) \leq \frac{1}{3},$$

and give examples to show that both extremes are possible.
Find corresponding bounds for $\mathsf{P}(A \cup B)$.


That's all Folks