Hypotheses. χ^2-scores and F-scores

Statistics

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DSBA 211
February 4, 2023

Seminar Overview

1. Quiz

2. χ^2-scores and F-scores

Quiz

The output voltage for a certain electric circuit is specified to be 130. A sample of 40 independent readings for the voltage gave a sample mean of 128.6 and a standard deviation of 2.1.

1. Test the hypothesis that the output voltage is 130 against the alternative that it is less than 130. Use 5% significance level.
2. What is the Probability of Type II error for the alternative $H_1 : \mu = 128$ for the decision rule, used in part 1?

Problem 1

An efficiency expert wishes to analyze the difference in productivity between workers exposed to two different lighting arrangements. She chooses a procedure in which the same 20 employees work under each arrangement for 1 week in turn. The outputs of each employee under the two lighting arrangements, LA1 and LA2, are summarized:

Num	LA1	LA2	Num	LA1	LA2
1	15	12	11	25	26
2	23	22	12	26	20
3	18	18	13	15	14
4	19	20	14	19	17
5	27	24	15	19	15
6	13	12	16	21	16
7	22	20	17	20	22
8	20	20	18	30	26
9	20	19	19	24	24
10	19	17	20	16	19

Problem 1

1. Find the mean and standard deviation of each of the 20-element set corresponding to LA1 and LA2 separately.
2. Run a two-sample hypothesis test with $H_0 : \mu_1 - \mu_2 = 0$, $H_1 : \mu_1 - \mu_2 > 0$, and $\alpha = 0.10$.
3. Find the p-value.
4. Form the set of 20 differences and find the mean and standard deviation.
5. Run a single sample hypothesis test with $H_0 : \mu_1 - \mu_2 = 0$, $H_1 : \mu_1 - \mu_2 > 0$, and $\alpha = 0.10$ on this set.
6. Find the p-value.
7. Are the observed differences in lighting arrangements significant?
8. Do the same analysis as above, comparing the results of the tests of two-sample and one-sample hypothesis at $\alpha = 0.05$, when alternative is $\mu_1 \neq \mu_2$.

Problem 2

A company produces electric devices operated by thermostatic control. The standard deviation of the temperature at which these controls actually operate should not exceed 2.0° Fahrenheit. For a random sample of twenty of these controls, the sample standard deviation of operating temperatures was 2.36° Fahrenheit. Stating any assumptions you need to make, test at the 5% significance level the null hypothesis that the population standard deviation is 2.0° against the alternative that it is bigger.

Problem 3

A random sample of ten students found the following figures, in hours, for time spent studying in the week before the final exams.

28 57 42 35 61 39 55 46 49 38

Assume that the population distribution is normal.

1. Find the sample mean and standard deviation.
2. Test at the 5% significance level the null hypothesis that the population mean is 40 hours against the alternative that it is higher.
3. Test at the 5% significance level against a two-sided alternative the null hypothesis that the population standard deviation is 10 hours.

Problem 4

The table below shows the annual salaries (in \$1000) of randomly selected doctors in public medical centers and private hospitals.

State	61	46	68	72	41	59	60	55	40
Private	72	77	54	28	57	24	82		

1. Test the null hypothesis that mean salary in private hospitals is \$1000 more than in public medical centers.
2. State carefully the assumptions you have made in the test in part 1.
3. Test the null hypothesis that the variances of the salaries in public medical centers and private hospitals are equal.

Problem 5

A firm A suggested for market a new coffee packing machine. The firm claims that the new model has higher precision than the old one. An old model has precision $\sigma = 5$ g. Independent expert was invited to test the statement of the producer. Six independent trials gave the following results:

weight (g)	102	103	97	99	100	97

1. At 5% significance level test the statement of the machine A producer.
2. Find 5% upper bound for the precision σ of the new model A.
3. Find 10% lower bound for the precision σ of the new model A.

Problem 5

4. Another firm B suggested its own model of the packing machine. It claims that model B is more precise than the new model of firm A. The expert tested model B as well, 8 trials gave the following results:

weight (g)	100	96	97	98	101	99	99	97

Is there enough statistical evidence to support the statement of the firm B?

Problem 6

Let X and Y be two random normal variables with distributions $\mathcal{N}\left(\mu_X, \sigma_X^2\right)$ and $\mathcal{N}\left(\mu_Y, \sigma_Y^2\right)$. Random sample of size 17 gives $\overline{x} = 100$, $s_X^2 = 9$, and a random sample of size 10 gives $\overline{y} = 105$, $s_Y^2 = 25$.

1. Find 95% upper bound for the ratio σ_Y/σ_X.
2. Find 95% lower bound for the ratio σ_Y/σ_X.
3. Test at 10% significance level $\mathrm{H}_0 : \mu_X = \mu_Y$.
4. Test at 10% significance level $\mathrm{H}_0 : \mu_X = \mu_Y - 3$.
5. If, in fact, $\sigma_X = \sigma_Y$, what is the probability that for the two random samples of sizes 17 and 10 $s_Y/s_X > 5/3$?

Look at the time!